

Recent Developments in Superstring Theory

hep-lat/0011073

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The talk contains a brief introduction string theory, followed by a discussion of some of the recent developments.

This talk will begin with a brief review of pre-1994 string theory. Then I shall describe the developments during 1994-1996 which include duality symmetries, D-branes and black holes. Finally I shall turn to the discussion of some of the developments during 1997-2000. The topics covered will include (M)atrix theory, Maldacena conjecture, non-commutative geometry, tachyon condensation, special limits of string theory in which gravity decouples (little string theories, OM theory etc.) and large radius compactification. This however is not an exhaustive review of string theory. Indeed, many of the important developments in this subject, like mirror symmetry, new ideas about cosmological constant, etc. will be left out of this review. I shall work in the convention $\hbar = 1, c = 1$.

1. String theory: pre-1994

As we all know, quantum field theory has been extremely successful in providing a description of elementary particles and their interactions. However, it does not work so well for gravity. The reason for this is ultraviolet divergences. In particular Feynman graphs involving gravitons, of the type shown in Fig.1, give divergent answers which cannot be made finite by standard renormalization procedure. String theory is an attempt to solve this problem.

The basic idea in string theory is quite simple[1,2]. According to this theory, different elementary particles which we observe in nature are different vibrational states of a one dimensional object, *i.e.* a string. In other words, instead of having different kinds of elementary particles, we

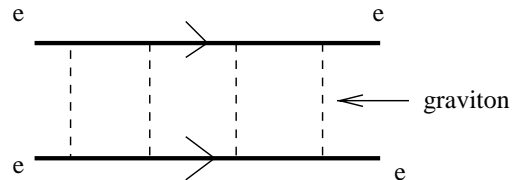


Figure 1. Ultraviolet divergent graphs in quantum gravity.

have only one kind of string, and the differences in the observed properties of elementary particles arise because they correspond to different quantum states of this string. Strings can be closed with no ends or open with suitable boundary condition at the two ends, as shown in Fig.2.

At the first sight, this would seem to contradict what we know about elementary particles like electrons and quarks: they behave like particles rather than one dimensional objects. However, when one estimates the typical size of a string, one finds that this size is of the order of $10^{-33}\text{cm} \sim (10^{19}\text{GeV})^{-1}$.² This is much smaller than the distance scale which can be probed by present day accelerators. Thus there is no immediate contradiction with experiments, since strings of such a small size will appear point-like in present day

²This size is controlled by the energy per unit length of the string, known as the string tension, which, in turn, is determined from the observed value of the Newton's constant of gravitation. As we shall see later, large radius compactification can drastically change the estimate of the string size, but it is still smaller than the distance scale that can be probed by current accelerators.

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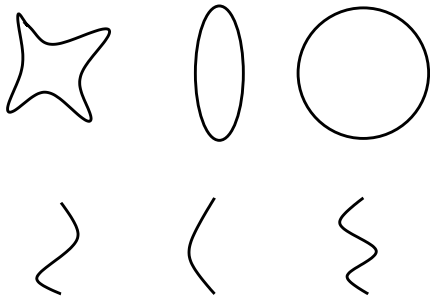


Figure 2. Different vibrational modes of closed and open strings.

experiments.

There is a well defined procedure for quantizing a string without violating Lorentz invariance. It turns out that

- Quantum string theories do not suffer from any ultraviolet divergence.
- Spectrum of a string theory contains a particle which has all the properties of a graviton – the mediator of gravitational interactions.

Thus string theory gives a finite quantum theory of gravity + other stuff.

There are however several problems with string theory. They are as follows:

- String theory is consistent only in (9+1) dimensional space-time instead of the (3+1) dimensional space-time in which we seem to live.
- Instead of a single consistent string theory, there are *five* consistent string theories in (9+1) dimensions. They are called type IIA, type IIB, type I, SO(32) heterotic, and $E_8 \times E_8$ heterotic string theories. On the other hand it is desirable that we have a single theory, as there is only one nature which string theory attempts to describe.

It turns out that the first problem is resolved via a procedure called compactification. The second

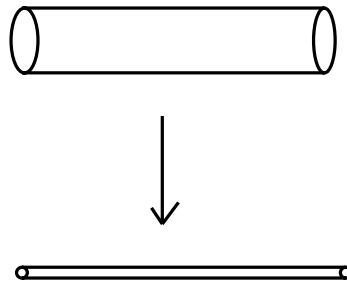


Figure 3. Compactification of a two dimensional theory to a one dimensional theory.

problem is partially resolved due to a property of string theory called duality; this will be discussed in the next section.

I shall now briefly discuss the idea of compactification. The idea in fact is quite simple: take 6 of the 9 spatial directions to be small and compact. As long as the sizes of the compact directions are smaller than the reach of the present day accelerators, the world will appear to be (3+1) dimensional. Since it is hard to draw a 9-dimensional space, a caricature of compactification has been shown in Fig.3 which demonstrates how a 2-dimensional space can be made to look like a one dimensional space. Here we take the two dimensional space to be the surface of a cylinder of radius R and infinite length. For large R (larger than the range of the most powerful telescope) the space will look like an ordinary two dimensional space of infinite extent in all directions. whereas for small R (smaller than the resolution of the most powerful microscope) the space will look one dimensional. This way an intrinsically two dimensional space can be made to look one dimensional. The same idea works in making an intrinsically 9 dimensional space look 3 dimensional.

It turns out that not all six dimensional compact manifolds can be used for this purpose, but there are many choices. A particularly important class of six dimensional manifolds, which can be used for string compactification, are known as Calabi-Yau manifolds. Thus each string theory

is (9+1) dimensions gives rise to many different string theories in (3+1) dimensions after compactification. Some of these theories come tantalizingly close to the observed universe[3]. In particular, they have:

- Gauge group containing $SU(3) \times SU(2) \times U(1)$
- Chiral fermions
- Three generations of quarks and leptons
- N=1 supersymmetry

etc. However, so far nobody has found a suitable compactification scheme which gives results in complete quantitative agreement with the observed universe (including masses of various elementary particles).

2. String Theory: 1994-1996

2.1. Duality in String Theory

Existence of duality symmetries in string theory started out as a conjecture and still remains a conjecture. However so many non-trivial tests of these conjectures have been performed by now that most people in the field are convinced of the validity of these conjectures[4–9].

A duality conjecture is a statement of equivalence between two or more apparently different string theories. Two of the most important features of a duality relation are as follows:

- Under a duality map, often an elementary particle in one string theory gets mapped to a composite particle in a dual string theory and vice versa. Thus classification of particles into elementary and composite loses significance as it depends on which particular theory we use to describe the system.
- Duality often relates a weakly coupled theory to a strongly coupled theory. If we denote by g and \tilde{g} the coupling constants in the two theories related by the duality map, then often there is a simple relation between them of the form:

$$g = (\tilde{g})^{-1}. \quad (2.1)$$

Thus a perturbation expansion in g contains information about non-perturbative

effects in \tilde{g} in the dual theory. In particular, tree level (classical) results in one theory is given by the sum of perturbative and non-perturbative results in the dual theory.

From this it should be clear that duality is a property of the quantum string theory and not its classical limit.

I shall now give some examples of duality:

- SO(32) heterotic string theory has been found to be dual to type I string theory in (9+1) dimensions.
- SO(32) (as well as $E_8 \times E_8$) heterotic string theory compactified on a four dimensional torus T^4 has been found to be dual to type IIA string theory compactified on a non-trivial four dimensional compact manifold called $K3$.
- Type IIB string theory has been found to be self dual, *i.e.* the theory at coupling constant g has been found to be equivalent to the same theory with coupling constant $1/g$.
- The SO(32) (as well as $E_8 \times E_8$) heterotic string theory, compactified on a six dimensional torus T^6 , has been found to be self-dual in the same sense.

Since duality relates different compactifications of different string theories, it provides us with a unified picture of all string theories. According to our new understanding based on duality, there is a single underlying theory – which I shall call the U -theory – with many degenerate vacua labelled by a set of parameters. Fig. 4 gives a schematic representation of this parameter space. In some special limits, the U -theory can be described by one of the (compactified) weakly coupled string theories. These special regions have been shaded in the figure. The rest of the regions represent string theories at finite / strong coupling. Note that besides the five corners labelling five weakly coupled string theories, there is another corner called M . This turns out to be a (10+1) dimensional theory, whose low energy limit is the (10+1) dimensional supergravity theory[10,8,11]. Often people use the same symbol M to describe this special corner as well as the whole region of the parameter space, but I shall

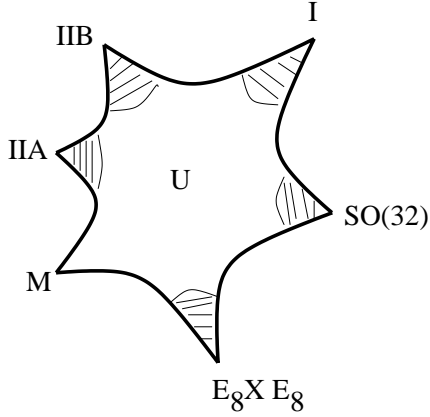


Figure 4. The parameter space of U/M theory.

reserve the name M for this special corner, and use the symbol U for the underlying theory with the full parameter space.

Thus we see that using the idea of duality, we can combine all string theories into a single theory, with a big parameter space. It turns out that these parameters themselves are dynamical, being related to vacuum expectation values of various fields. The ultimate dream of a string theorist is that when we fully understand the dynamics of string theory, we shall find some dynamical principle which determines a unique point in this parameter space, and that this point will correctly describe the nature that we see.

2.2. D-branes

Dirichlet(D)-branes are soliton like configurations in type IIA/IIB/I string theories[12,13]. However, the description of D-branes is quite different from the way we normally describe a conventional soliton. To understand this distinction, let us review the way a conventional soliton is described in a quantum field theory:

1. First we construct a time independent solution of the classical equations of motion with energy density localised around a p -dimensional spatial hypersurface. We shall call such a solution a p -brane soliton. Thus for example a 0-brane corresponds to a par-

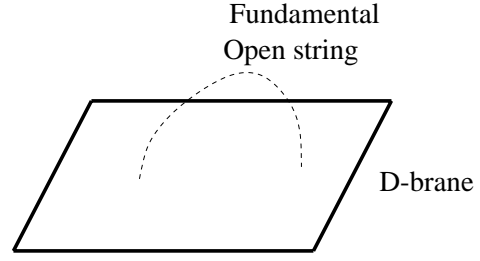


Figure 5. Open string excitations on a D-brane.

ticle like soliton, a 1-brane corresponds to a string like soliton, a 2-brane corresponds to a membrane like soliton and so on.

2. Next we identify the fluctuation modes of various fields around this background solution which are *localised* around the brane. These modes describe the fluctuation of the brane around its equilibrium position, and include for example oscillation modes of the brane in directions transverse to the brane.
3. We then construct the $(p + 1)$ dimensional field theory which describes the dynamics of these modes. (This can be derived from the original field theory whose soliton we are analysing.) This field theory describes the classical dynamics of the soliton solution.
4. In order to study the quantum dynamics of this soliton, we quantize this field theory.

In describing a D-brane we start from step 4 and work backwards.

I shall now give explicit description of D-branes in type IIA and type IIB string theories. Elementary excitations in type IIA / IIB string theories are *closed strings*. D- p -branes in these theories are p -dimensional soliton like objects whose quantum dynamics is described by the theory of *open strings* whose ends are constrained to move on the D-brane. This has been illustrated in Fig.5. Starting from this description we can work backwards and derive all the properties of these branes. It turns out that type IIB string theory has stable (BPS) D- p -branes for p -odd (1, 3, 5, 7,

9), whereas type IIA string theory has stable D- p -branes for p -even (0, 2, 4, 6, 8). In particular, type IIB string theory contains a D9-brane which fills all space. For this theory one can also add to this list the $p = -1$ brane, representing a configuration that is localised not only in all space directions but also in the time direction. This is called the D-instanton.

Under duality often elementary closed string states in one theory get mapped to D-brane states in the dual theory. Indeed, this is how the importance of D-branes in the study of non-perturbative string theory was first realised[13]. But since their discovery D-branes have found many applications in string theory.

A particularly important property of D-branes is as follows[14]. *The $(p+1)$ dimensional effective field theory, describing the dynamics of N coincident D- p -branes at low energy, is a supersymmetric $U(N)$ gauge theory with 16 supercharges.* This is an ordinary gauge theory with a set of Majorana fermions and scalars in the adjoint representation of the gauge group, with the Yukawa and scalar self-couplings determined in terms of the gauge coupling via specific relations. We shall encounter the (3+1)-dimensional version of this theory again later while discussing the Maldacena conjecture.

2.3. Black Hole Entropy and Hawking Radiation

Black holes are classical solutions of general relativity. They can be formed from collapse of matter under gravitational pull. Classically a black hole is completely black, *i.e.* it absorbs everything that falls in without emitting anything. But this picture changes significantly in the quantum theory. It turns out that in the quantum theory, black holes behave as perfect black bodies at *finite temperature* proportional to the inverse mass of the black hole. In particular,

- black holes emit thermal radiation known as Hawking radiation, and
- a black hole carries an entropy proportional to its surface area, known as the Bekenstein-Hawking entropy.

These results were derived using a semi-classical analysis, but there was no microscopic

(statistical) understanding of this entropy and radiation in this semi-classical treatment. In particular, the expressions for the entropy and the Hawking temperature were given in terms of classical geometrical properties of the space-time around the black hole. It was as if we had thermodynamics without statistical mechanics. This led Hawking to suggest that starting with a collapsing spherical shell of matter in a pure quantum state, and letting it collapse into a black hole and subsequently evaporate via Hawking radiation, we can have a pure quantum state get transformed to a mixed state described by a thermal density matrix. Such a process violates the laws of quantum mechanics.

It turns out that in string theory, for a special class of black holes, one can find dual descriptions as:

- a classical solution of the equations of motion, and
- a configuration of D-branes.

Thus for these black holes we can compute entropy and Hawking radiation in two ways:

- Use Bekenstein and Hawking's formulæ to compute the entropy and the rate of radiation from the classical solution.
- Use the quantum string theory living on the D-brane to compute the number of quantum states N and the rate of radiation from the system. From this we can compute the entropy by taking the logarithm of the degeneracy N .

These two computations give identical result[15–18].³ Thus this analysis provides a microscopic explanation of the black hole entropy and Hawking radiation for these special class of black holes. It remains to be seen whether these results generalize to the more general black holes.

3. String Theory: 1997-2000

The developments in string theory during this period can be described as study of various as-

³The two computations are valid in different regions of the parameter space, and we need to continue the result computed in one region to the other region by invoking supersymmetric non-renormalization theorems.

pects of U -theory. I shall discuss some of them here.

3.1. Matrix Theory

Note that one corner of the parameter space of U -theory is M -theory. M -theory is known to reduce to 11-dimensional supergravity theory at low energy. But there is no known systematic procedure for computing corrections to this low energy approximation. *Matrix theory*[19] is a proposal for defining M -theory beyond this low energy approximation. Although I shall not describe the logic which led to the formulation of Matrix-theory, let me just state that this proposal comes from examining the dynamics of D0-branes in IIA in the appropriate limit in which it approaches the M -theory corner[20–22].

The Matrix theory proposal is as follows. Consider a scattering process in M -theory: $A + B \rightarrow C + D + \dots$ for a set of external states A, B, C, D, \dots . Let us go to a frame in which the centre of mass has an infinite boost. This is known as the infinite momentum frame. *According to Matrix theory, M -theory in the infinite momentum frame is equivalent to a quantum mechanical system.* The dynamical variables in this quantum mechanical system are $N \times N$ matrices where N is proportional to the total momentum of the original system. Since in the infinite momentum frame the total momentum of any system is infinite, we need to take the $N \rightarrow \infty$ limit at the end.

This, in principle, gives an algorithm for computing any physical quantity in M -theory by mapping it to an appropriate quantity in this quantum mechanical system. We can perform various consistency checks. In particular at low energy, scattering amplitudes computed using matrix quantum mechanics must agree with those computed from 11-dimensional supergravity. Matrix theory has passed many such tests[23,24]. Of course, in principle one should be able to use matrix theory to go beyond tree level supergravity. This has not been achieved so far.

3.2. Maldacena Conjecture

The starting point here is the study of N coincident D3-branes in type IIB string theory in the large N limit. In this limit the system has dual descriptions

- as a solution of the classical equations of

motion of string theory / supergravity, and

- as a D-brane system.

Requiring that these two descriptions are equivalent led Maldacena to the following conjecture[25]: *Type IIB string theory on $(AdS)_5 \times S^5$ is equivalent to $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory in $(3+1)$ dimensions.*

There are several terms in the above statement which may not be familiar, so I shall now define them. S^5 is an ordinary five dimensional sphere of radius R defined by the equation

$$\sum_{i=1}^6 (y_i)^2 = R^2, \quad (3.1)$$

where y_i are the coordinates of a six dimensional Euclidean space. $(AdS)_5$ is the five dimensional anti-de Sitter space described by the equation

$$(x_0)^2 + (x_1)^2 - (x_2)^2 - (x_3)^2 - (x_4)^2 - (x_5)^2 = R^2, \quad (3.2)$$

where x_i 's describe a six dimensional space with metric

$$ds^2 = -(dx_0)^2 - (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2. \quad (3.3)$$

It turns out that the boundary of $(AdS)_5$ is a $(3+1)$ dimensional Minkowski space. Often in order to make precise statements one needs to Wick rotate to the Euclidean version of AdS_5 . This is obtained by changing the signs of the $(x_1)^2$ term in eq.(3.2) and the $(dx_1)^2$ term in eq.(3.3). Its boundary is a four dimensional Euclidean space.

Finally we need to define $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory. This is an ordinary $SU(N)$ gauge theory with 6 scalars and 4 Majorana fermions in the adjoint representation of the gauge group, with specific relations between the gauge coupling constant and various Yukawa and scalar self-couplings. In particular all the Yukawa and scalar self-couplings are determined in terms of the gauge coupling. This theory turns out to be a conformally invariant $(3+1)$ dimensional field theory, *i.e.* the β -functions in this theory vanish.

According to the Maldacena conjecture, the relation between the dimensionless parameters of IIB string theory on $AdS_5 \times S^5$ and supersymmetric $SU(N)$ Yang-Mills theory is as follows:

$$g_{string} = g_{YM}^2, \quad R = (4\pi g_{YM}^2 N)^{1/4}, \quad (3.4)$$

where g_{YM} and g_{string} denote the coupling constants of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and the type IIB string theory respectively, R is the radius of S^5 and AdS_5 as introduced through eqs.(3.1) and (3.2), and N is the N of $SU(N)$.

We are now in a position to state the precise form of the Maldacena conjecture[27,28]: *There is a one to one correspondence between the physical Greens functions in type IIB string theory on $(AdS_5 \times S^5)$ and the correlation functions of gauge invariant operators in $\mathcal{N} = 4$ supersymmetric $SU(N)$ gauge theory on the boundary of $(AdS)_5$, with the identification of the parameters as given in eq.(3.4).*

Consider now the 't Hooft large N limit[26]:

$$g_{YM} \rightarrow 0, \quad N \rightarrow \infty, \quad \lambda \equiv g_{YM}^2 N \text{ fixed.} \quad (3.5)$$

This gives, using eq.(3.4)

$$g_{string} \rightarrow 0, \quad R = (4\pi\lambda)^{1/4} \text{ fixed.} \quad (3.6)$$

Thus if λ is large then R is large. In this limit the smallness of g_{string} implies that we can ignore the string loop corrections, *i.e.* restrict ourselves to string tree diagrams or classical string theory. The largeness of R on the other hand allows us to use the low energy approximation to string theory, which is type IIB supergravity theory. Thus in this case, quantum string theory can be approximated by classical supergravity.

Many other examples of this kind of relationship have been found by studying other brane systems[25]. The generic form that such a relation takes is that a string theory / M -theory on a manifold K is equivalent to a quantum field theory on the boundary of K . The precise form of this quantum field theory depends on the choice of K and the particular string theory / M -theory that we are using.

There have been various applications of Maldacena conjecture, both for using supergravity / string theory to study strong coupling limit of gauge theories, and using gauge theory to study non-perturbative aspects of string theory. Here I shall discuss one application of each kind. It had been conjectured earlier that *in a consistent quantum theory of gravity, the fundamental degrees of freedom reside at the boundary of space-time and not in the interior. Furthermore,*

*there is ~ 1 degree of freedom per Planck 'area'.*⁴ This principle is known as the holographic principle[29,30]. Maldacena conjecture provides a concrete verification of the holographic principle for type IIB string theory on $(AdS)_5 \times S^5$ by relating string theory on $AdS_5 \times S^5$ to a gauge theory living on its boundary. In the $\mathcal{N} = 4$ supersymmetric gauge theory described earlier, it is straightforward to count the total number of degrees of freedom after a suitable ultra-violet regularisation of the gauge theory. It turns out that this ultra-violet cut-off is related to the infrared cut-off of the string theory on $AdS_5 \times S^5$, so that with this cut-off in place, the boundary of AdS_5 at a given instant of time, instead of having infinite volume like an ordinary 3 dimensional space, has finite volume. Taking the ratio of the number of degrees of freedom of the gauge theory and the volume of this 3-dimensional space, we find that the cut-off dependence goes away, and there is indeed one (up to a numerical factor) degree of freedom per Planck volume living on the boundary of AdS_5 [31].

Another application of the Maldacena conjecture has been in the study of renormalization group (RG) flows in conformal field theories[32–34]. For example, we can perturb the $\mathcal{N} = 4$ superconformal field theory by adding some relevant perturbation. The resulting theory either

- flows to a conformal field theory, or
- flows to a theory with a mass gap (*e.g.* a confining theory).

On the supergravity side this perturbed theory should be described by a solution which in some region of space-time (UV) looks like $AdS_5 \times S^5$ background, but in another region (IR) differs from the $AdS_5 \times S^5$ geometry and represents the infra-red theory. The situation has been illustrated in Fig.6. Thus the RG evolution parameter becomes a coordinate in space time. For all RG flows which can be described via the supergravity configurations of this type, the c -theorem (which states that the central charge of a conformal field theory decreases along the RG flow) follows naturally from general properties of the supergravity equations of motion. This procedure has also

⁴Note that if the boundary is an n -dimensional space, then the area refers to the n -volume of this space.

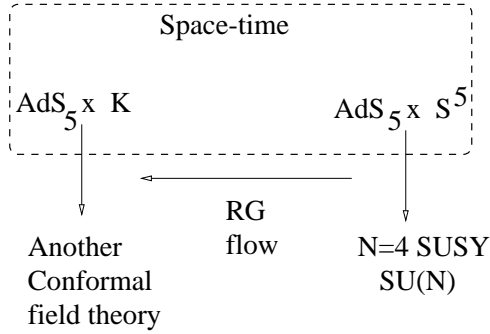


Figure 6. Renormalization group flow and solutions of supergravity equations of motion. The upper part of the diagram shows the space-time picture of the supergravity solution. To the far right (the UV region) the space-time looks like a piece of $AdS_5 \times S^5$, whereas to the far left (IR region) the space-time looks like a piece of $AdS_5 \times K$ for some manifold K . The lower part shows the interpretation of different regions of the supergravity solution as quantum field theories.

been used to describe RG flow to theories with mass gap and confinement, where in the infrared the theory does not flow to a conformal field theory[35]. In these cases space-time at the far left end (representing infrared direction in RG flow) has a more complicated structure[35].

3.3. D-branes and Non-commutative Field Theories

The dynamics of a D- p -brane is described by an open string theory, which could be regarded as a $(p+1)$ -dimensional field theory with infinite number of fields. In the low energy limit only a finite number of massless fields survive whose dynamics is described by a $(p+1)$ -dimensional effective quantum field theory. One of these fields is a $U(1)$ gauge field. We can study this $(p+1)$ -dimensional field theory in the presence of constant background electric / magnetic field associated to this $U(1)$ gauge field. It turns out that if we switch on a constant background magnetic field, and take a suitable limit involving the string tension, the string coupling constant, the background metric

and the background magnetic field, then the dynamics of the D-brane in this limit is described by a non-commutative gauge theory[36–39]. The action for this non-commutative gauge theory is obtained by starting with the original low energy effective action that we had for zero magnetic field background, and then *replacing all products of fields in the original action by the non-commutative product*:

$$\begin{aligned} A(x)B(x) &\rightarrow A(x) * B(x) \\ &= \exp(\Theta^{\mu\nu} \partial_\mu \partial'_\nu) A(x)B(x')|_{x=x'}, \end{aligned} \quad (3.7)$$

where $\Theta^{\mu\nu}$ is an anti-symmetric matrix determined in terms of the strength of the background magnetic field and other parameters. Thus one can use non-commutative field theory results to study D-branes in the presence of background magnetic field and vice-versa.

One of the important results coming out of these studies is that *the infrared and ultraviolet effects do not decouple in a non-commutative field theory*[40]. Thus physics at large distance can affect physics at short distance and vice versa. There have been suggestions that this is also a general property of string theory[41]. Another important result is the discovery that non-commutative scalar field theories can contain stable soliton solutions[42] even if their commutative counterparts do not contain such solutions due to Derrick's theorem[43].

3.4. Tachyon Condensation on Brane-Antibrane System

As has been discussed earlier, type IIB string theory has stable D p -branes for odd p . In particular, $p = 9$ describes a space-filling 9-brane. This theory also has stable D \bar{p} -branes for odd p . These are D p -branes with opposite orientation. It turns out that although D p -branes and D \bar{p} -branes are individually stable, a system of coincident D p -brane D \bar{p} -brane has tachyonic modes. These are scalar fields T with negative mass². Thus such a system is classically unstable.

The question that arises naturally is: Is there a stable minimum of the *classical* tachyon potential $V(T)$? The conjectured answer to this question is as follows[44]:

1. The minimum of $V(T)$, $T = T_0$, describes

‘nothing’ where the original energy of the brane-antibrane system is exactly cancelled by $V(T_0)$

2. There are no perturbative open string excitations around this vacuum, but there are solitons. These solitons describe lower dimensional D-branes.

Evidence for these conjectures come from various sources (string field theory[45–51], conformal field theory[52,53], non-commutative field theory[42,54,55], Maldacena conjecture[56], Matrix theory[57,58] etc.) There are several applications of these results:

- Describing all D-branes as solitons on space-filling branes gives a way to classify all possible D-brane configurations in a string theory in terms of *K-theory*[59,60].
- These results suggest that the open string theory on the space-filling brane anti-brane system may provide a non-perturbative formulation of string theory, since
 - it naturally contains all D-branes as solitons, and
 - there is also evidence that this theory contains closed string states[55,61–64].

From this viewpoint, the fundamental degrees of freedom are open strings, and closed strings (including gravitons) and D-branes are composite objects.

3.5. Special Limits of String Theory: String Theories without Gravity

If we have a p -brane soliton, then the degrees of freedom on the soliton describe a quantum theory living in $(p + 1)$ dimensions. Typically these degrees of freedom interact with those living in the bulk, and we do not have a consistent quantum theory involving only the degrees of freedom living on the p -brane. But in certain special limits, the degrees of freedom in the bulk may decouple. In that case, we get a consistent $(p + 1)$ -dimensional quantum theory on the p -brane world-volume *without gravity* since gravity lives in the bulk. One such limit is the low energy limit in which we recover a quantum field theory living on the p -brane world-volume. As discussed

in subsection 3.3, a variation of this limit can also give rise to non-commutative field theories. But there are other limits which give us full fledged string theory (or other quantum theories which cannot be described as quantum field theories) in $(p + 1)$ -dimensions without gravity. These theories are expected to capture many of the important features of standard string theories, without being plagued by the conceptual problems which arise due to the presence of gravity. I shall give a few examples here.

- LST (Little string theories): Besides containing the D-brane solitons, string theories also contain another kind of 5-brane soliton, known as the NS 5-brane. In the presence of a set of coincident NS 5-branes, we have a set of degrees of freedom localised on the 5-branes, describing the dynamics of these NS 5-branes, and another set of degrees of freedom living in the $(9+1)$ dimensional bulk space-time. It turns out that in the limit of vanishing string coupling constant with the string tension remaining finite, the degrees of freedom living on the NS 5-branes decouple from the degrees of freedom living in the bulk, and hence we get a consistent $(5+1)$ dimensional string theory without gravity[65–67]. This theory and its various cousins obtained via compactification are known as little string theories.
- NCOS (Non-commutative open string) theory: In this case the starting point is a $D-p$ -brane in the presence of a non-zero constant electric field background. By taking an appropriate limit involving the string coupling constant, the string tension, the background metric and the electric field strength on the $D-p$ -brane one can decouple the bulk modes from the modes living on the $D-p$ -brane[68,69]. The $(p + 1)$ dimensional theory obtained this way has been called non-commutative open string theories, since the fundamental degrees of freedom in this theory are open strings, but the action involving these open strings is related to the action of the usual open string theory by a replacement of all ordinary products by appropriate non-commutative products.

- OM (open membrane) theory: Like string theory, M -theory also has solitonic branes. In particular, it has a 5-brane and a 2-brane soliton solutions. Among the degrees of freedom living on the 5-brane world-volume, there is a rank 2 anti-symmetric tensor gauge field. Its field strength is a totally anti-symmetric self-dual rank 3 tensor in $(5+1)$ dimensions. It turns out that starting from a configuration with constant non-vanishing field strength of this anti-symmetric tensor field, and taking an appropriate limit involving the Planck mass, the background metric and this field strength, one can decouple the bulk modes from the brane modes[70]. The resulting $(5+1)$ dimensional quantum theory living on the 5-brane has been called the open membrane theory. As the name suggests, excitations in this theory include open membranes with their boundaries stuck to the 5-brane.
- OD p (Open D p -brane) theories: As in the case of little string theories, here the starting point is an NS 5-brane of type IIA string theory. But instead of putting it in trivial background space-time, we now switch on constant background values of some appropriate bulk fields (known as Ramond-Ramond (RR) gauge fields). By taking appropriate limits of the string tension, the string coupling constant, the background metric and the background value of the RR gauge fields, one can again decouple the theory in the bulk from the theory on the brane[70]. The result is a quantum theory on the 5-brane, known as open D- p -brane theories. The name originates from the fact that the excitations on the brane include open D- p -branes (the value of p depends on which particular RR gauge field is switched on) with their boundaries stuck on the 5-brane.

3.6. Large Radius Compactification

In conventional compactification of string theory leading to semi-realistic models, both gravity and gauge fields come from the closed string sector. In this scheme, a direct upper bound on the

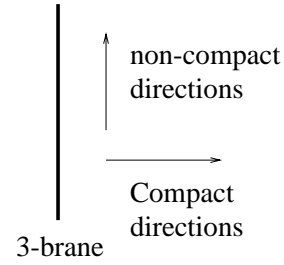


Figure 7. Compactification with branes.

size of the compact dimension comes from experimental verification of various force laws to small distance scales. Thus, for example, test of QED down to a distance scale of $(TeV)^{-1}$ will mean that the size of the compact direction cannot be larger than $(TeV)^{-1}$. Otherwise we would have to use higher dimensional QED for our computation.

Discovery that gauge fields can live on D-branes has given rise to new possibilities[71–75]. Consider for example the scenario shown in Fig.7 where 6 of the directions are compactified on a manifold K , and there is a set of three branes with their world-volume directed along the non-compact directions, situated at a given point in the compact space. All gauge fields and known matter fields come from the world-volume theory on the three brane, and gravity comes from the closed strings living in the bulk. Thus here gauge fields are always $(3+1)$ dimensional, irrespective of the size of the compact dimensions. In this scenario, the only direct upper bound on the size of the compact dimension comes from the test of inverse square law for gravity at short distance scale. This gives an upper bound of about a millimetre on the size of the compact dimensions. There are of course other indirect bounds which we shall not discuss here, but the fact remains that the size of these extra compact dimensions can be much larger than $(TeV)^{-1}$, – the distance scale probed in the present accelerator experiments.

Presence of large extra dimensions also changes the relation between four dimensional Planck

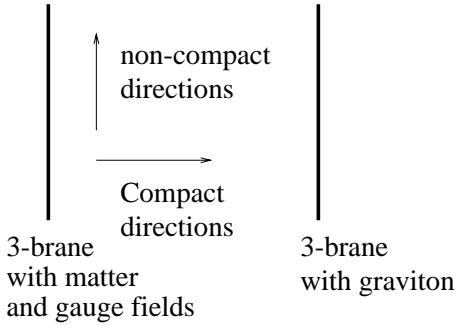


Figure 8. Illustration of the Randall-Sundrum scenario.

scale and string scale. The new relation is of the form:

$$M_{Planck} \sim M_{string} \sqrt{M_{string}^6 V_{compact}}, \quad (3.8)$$

where M_{Planck} denotes the four dimensional Planck mass, M_{string} denotes the square root of the string tension, and $V_{compact}$ denotes the volume of the compact six dimensional manifold. (We have taken the string coupling to be of order 1). From this relation we see that if the size of the compact direction is much larger than the string scale, *i.e.* if $M_{string}(V_{compact})^{1/6} \gg 1$, then $M_{string} \ll M_{Planck}$. It has even been suggested that M_{string} can be of the order of a TeV; this requires $M_{string}(V_{compact})^{1/6}$ to be of order $10^5 - 10^6$. This would resolve the usual hierarchy problem as to why M_{string} is so large compared to the mass scale of weak interaction – in this scheme they are of the same order. However, it gives rise to a new hierarchy problem, – that of explaining why $M_{string}(V_{compact})^{1/6}$ is so large. Various ideas have been put forth, but there is no definite conclusion.

A twist to this tale is provided by the Randall-Sundrum scenario for string compactification[76, 77] illustrated in Fig.8. In this scenario, the graviton that we observe in four dimension is not the usual ten dimensional bulk graviton carrying zero momentum in the compact direction, but a particular mode of the ten dimensional graviton which is localised on a brane. This brane how-

ever is not the brane that we live on, but another brane (which we shall call the gravity brane), separated from us by a fairly large distance along the compact direction. Since the wave-function of the graviton falls off exponentially away from the gravity 3-brane, it has a very small value on the matter 3-brane. This fact can be used to explain why gravity couples so weakly to matter, *i.e.* why the effective Planck mass in our (3+1) dimensional world is so large. The advantage of the scenario is that due to the exponential fall off of the graviton wave-function away from the gravity brane, one can generate a large hierarchy between the effective four dimensional Planck scale and the weak scale without having to make the actual separation between the gravity brane and our brane very large.

4. Summary

I shall end by summarising the main points once more.

- String theory has had reasonable success in providing a consistent quantum theory of gravity. This includes:
 1. Finiteness of perturbation theory
 2. Partial resolution of the problems associated with quantum mechanics of black holes
 3. Explicit realization of holographic principle in certain backgrounds
- String theory also has the potential for providing a unified theory of all interactions. In particular, it can give rise to
 1. Gauge group containing $SU(3) \times SU(2) \times U(1)$
 2. Chiral fermions
 3. Three generations of quarks and leptons
 4. N=1 supersymmetry
- It has also proved to be an internally consistent and beautiful theory. In particular, string duality provides
 1. Unification of all string theories.

- 2. Democracy between elementary and composite particles.
- 3. Unification of classical and quantum effects.
- Progress in string theory has also dramatically improved our understanding of various aspects of quantum field theories and other quantum theories based on extended objects. New relationship between quantum field theories and string theories have been discovered during the past few years.
- The last few years have seen several attempts at giving a non-perturbative definition of string/M(U) theory. It is still too early to say if any of them will give rise to a fruitful approach to the study of non-perturbative effects in string theory.
- The brane world scenario has given rise to novel possibilities for string compactification.

However, despite this enormous progress, string theory is still far from achieving its final goal, which is to provide a unified theory of all matter and their interactions.

Acknowledgment: I wish to thank S. Rao for comments on the manuscript.

REFERENCES

1. M. B. Green, J. H. Schwarz and E. Witten, "Superstring Theory. Vol. 1 and 2," *Cambridge, Uk: Univ. Pr. (1987) (Cambridge Monographs On Mathematical Physics)*.
2. J. Polchinski, "String Theory, Vol.1 and 2", *Cambridge, Uk: Univ. Pr. (1998) (Cambridge Monographs On Mathematical Physics)*.
3. P. Candelas, G. T. Horowitz, A. Strominger and E. Witten, "Vacuum Configurations For Superstrings," Nucl. Phys. **B258**, 46 (1985).
4. A. Font, L. Ibanez, D. Lust and F. Quevedo, Phys. Lett. **B249** (1990) 35.
5. A. Sen, "Strong - weak coupling duality in four-dimensional string theory," Int. J. Mod. Phys. **A9**, 3707 (1994) [hep-th/9402002].
6. J. H. Schwarz, "Evidence for nonperturbative string symmetries," Lett. Math. Phys. **34**, 309 (1995) [hep-th/9411178].
7. C. M. Hull and P. K. Townsend, "Unity of superstring dualities," Nucl. Phys. **B438**, 109 (1995) [hep-th/9410167].
8. E. Witten, "String theory dynamics in various dimensions," Nucl. Phys. **B443**, 85 (1995) [hep-th/9503124].
9. S. Kachru and C. Vafa, "Exact results for N=2 compactifications of heterotic strings," Nucl. Phys. **B450**, 69 (1995) [hep-th/9505105].
10. P. K. Townsend, "The eleven-dimensional supermembrane revisited," Phys. Lett. **B350**, 184 (1995) [hep-th/9501068].
11. J. H. Schwarz, "Lectures on superstring and M theory dualities," Nucl. Phys. Proc. Suppl. **55B**, 1 (1997) [hep-th/9607201].
12. J. Dai, R. G. Leigh and J. Polchinski, "New Connections Between String Theories," Mod. Phys. Lett. **A4**, 2073 (1989).
13. J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," Phys. Rev. Lett. **75**, 4724 (1995) [hep-th/9510017].
14. E. Witten, "Bound States Of Strings And p-Branes," Nucl. Phys. **B460**, 335 (1996) [hep-th/9510135].
15. A. Strominger and C. Vafa, "Microscopic Origin of the Bekenstein-Hawking Entropy," Phys. Lett. **B379**, 99 (1996) [hep-th/9601029].
16. C. G. Callan and J. M. Maldacena, "D-brane Approach to Black Hole Quantum Mechanics," Nucl. Phys. **B472**, 591 (1996) [hep-th/9602043].
17. S. R. Das and S. D. Mathur, "Comparing decay rates for black holes and D-branes," Nucl. Phys. **B478**, 561 (1996) [hep-th/9606185].
18. J. Maldacena and A. Strominger, "Black hole greybody factors and D-brane spectroscopy," Phys. Rev. **D55**, 861 (1997) [hep-th/9609026].
19. T. Banks, W. Fischler, S. H. Shenker and L. Susskind, "M theory as a matrix model: A conjecture," Phys. Rev. **D55**, 5112 (1997) [hep-th/9610043].
20. M. R. Douglas, D. Kabat, P. Pouliot and S. H. Shenker, "D-branes and short distances in string theory," Nucl. Phys. **B485**, 85 (1997) [hep-th/9608024].
21. A. Sen, "D0 branes on T(n) and matrix the-

- ory,” Adv. Theor. Math. Phys. **2**, 51 (1998) [hep-th/9709220].
22. N. Seiberg, “Why is the matrix model correct?,” Phys. Rev. Lett. **79**, 3577 (1997) [hep-th/9710009].
 23. K. Becker and M. Becker, “A two-loop test of M(atr)ix theory,” Nucl. Phys. **B506**, 48 (1997) [hep-th/9705091].
 24. K. Becker, M. Becker, J. Polchinski and A. Tseytlin, “Higher order graviton scattering in M(atr)ix theory,” Phys. Rev. **D56**, 3174 (1997) [hep-th/9706072].
 25. J. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. **2**, 231 (1998) [hep-th/9711200].
 26. G. ’t Hooft, “A Planar Diagram Theory For Strong Interactions,” Nucl. Phys. **B72**, 461 (1974).
 27. S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” Phys. Lett. **B428**, 105 (1998) [hep-th/9802109].
 28. E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].
 29. C. R. Stephens, G. ’t Hooft and B. F. Whiting, “Black hole evaporation without information loss,” Class. Quant. Grav. **11**, 621 (1994) [gr-qc/9310006].
 30. L. Susskind, “The World as a hologram,” J. Math. Phys. **36**, 6377 (1995) [hep-th/9409089].
 31. L. Susskind and E. Witten, “The holographic bound in anti-de Sitter space,” hep-th/9805114.
 32. L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, “Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics,” JHEP **9812**, 022 (1998) [hep-th/9810126].
 33. J. Distler and F. Zamora, “Non-supersymmetric conformal field theories from stable anti-de Sitter spaces,” Adv. Theor. Math. Phys. **2**, 1405 (1999) [hep-th/9810206].
 34. D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, “Renormalization group flows from holography supersymmetry and a c-theorem,” hep-th/9904017.
 35. J. Polchinski and M. J. Strassler, “The string dual of a confining four-dimensional gauge theory,” hep-th/0003136.
 36. A. Connes, M. R. Douglas and A. Schwarz, “Noncommutative geometry and matrix theory: Compactification on tori,” JHEP **9802**, 003 (1998) [hep-th/9711162].
 37. M. R. Douglas and C. Hull, “D-branes and the noncommutative torus,” JHEP **9802**, 008 (1998) [hep-th/9711165].
 38. V. Schomerus, “D-branes and deformation quantization,” JHEP **9906**, 030 (1999) [hep-th/9903205].
 39. N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP **9909**, 032 (1999) [hep-th/9908142].
 40. S. Minwalla, M. Van Raamsdonk and N. Seiberg, “Noncommutative perturbative dynamics,” hep-th/9912072.
 41. T. Banks, “Cosmological breaking of supersymmetry or little Lambda goes back to the future II,” hep-th/0007146.
 42. R. Gopakumar, S. Minwalla and A. Strominger, “Noncommutative solitons,” JHEP **0005**, 020 (2000) [hep-th/0003160].
 43. G. H. Derrick, “Comments On Nonlinear Wave Equations As Models For Elementary Particles,” J. Math. Phys. **5**, 1252 (1964).
 44. A. Sen, “Non-BPS states and branes in string theory,” hep-th/9904207.
 45. A. Sen and B. Zwiebach, “Tachyon condensation in string field theory,” JHEP **0003**, 002 (2000) [hep-th/9912249].
 46. N. Berkovits, “The tachyon potential in open Neveu-Schwarz string field theory,” JHEP **0004**, 022 (2000) [hep-th/0001084].
 47. N. Moeller and W. Taylor, “Level truncation and the tachyon in open bosonic string field theory,” Nucl. Phys. **B583**, 105 (2000) [hep-th/0002237].
 48. N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP **0008**, 039 (2000) [hep-th/0005036].
 49. A.A. Gerasimov and S.L. Shatashvili, *On exact tachyon potential in open string field theory*, hep-th/0009103.
 50. D. Kutasov, M. Marino and G. Moore, *Some exact results on tachyon condensation*

- in string field theory*, hep-th/0009148; “Remarks on tachyon condensation in superstring field theory,” hep-th/0010108.
51. D. Ghoshal and A. Sen, “Normalisation of the background independent open string field theory action,” hep-th/0009191.
 52. A. Sen, “SO(32) spinors of type I and other solitons on brane-antibrane pair,” JHEP **9809**, 023 (1998) [hep-th/9808141].
 53. J. A. Harvey, D. Kutasov and E. J. Martinec, “On the relevance of tachyons,” hep-th/0003101.
 54. K. Dasgupta, S. Mukhi and G. Rajesh, “Non-commutative tachyons,” JHEP **0006**, 022 (2000) [hep-th/0005006].
 55. J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, “D-branes and strings as non-commutative solitons,” JHEP **0007**, 042 (2000) [hep-th/0005031].
 56. N. Drukker, D. J. Gross and N. Itzhaki, “Sphalerons, merons and unstable branes in AdS,” hep-th/0004131.
 57. P. Kraus, A. Rajaraman and S. Shenker, “Tachyon condensation in noncommutative gauge theory,” hep-th/0010016.
 58. G. Mandal and S. R. Wadia, “Matrix model, noncommutative gauge theory and the tachyon potential,” hep-th/0011094.
 59. E. Witten, “D-branes and K-theory,” JHEP **9812**, 019 (1998) [hep-th/9810188].
 60. P. Horava, “Type IIA D-branes, K-theory, and matrix theory,” Adv. Theor. Math. Phys. **2**, 1373 (1999) [hep-th/9812135].
 61. P. Yi, “Membranes from five-branes and fundamental strings from Dp branes,” Nucl. Phys. **B550**, 214 (1999) [hep-th/9901159].
 62. O. Bergman, K. Hori and P. Yi, “Confinement on the brane,” Nucl. Phys. **B580**, 289 (2000) [hep-th/0002223].
 63. G. Gibbons, K. Hori and P. Yi, “String fluid from unstable D-branes,” hep-th/0009061.
 64. A. Sen, “Fundamental strings in open string theory at the tachyonic vacuum,” hep-th/0010240.
 65. N. Seiberg, “New theories in six dimensions and matrix description of M-theory on T^{*5} and $T^{*5}/Z(2)$,” Phys. Lett. **B408**, 98 (1997) [hep-th/9705221].
 66. A. Losev, G. Moore and S. L. Shatashvili, “M & m’s,” Nucl. Phys. **B522**, 105 (1998) [hep-th/9707250].
 67. O. Aharony, “A brief review of ‘little string theories’,” Class. Quant. Grav. **17**, 929 (2000) [hep-th/9911147].
 68. N. Seiberg, L. Susskind and N. Toumbas, “Strings in background electric field, space/time noncommutativity and a new noncritical string theory,” JHEP **0006**, 021 (2000) [hep-th/0005040].
 69. R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, “S-duality and noncommutative gauge theory,” JHEP **0006**, 036 (2000) [hep-th/0005048].
 70. R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, “OM theory in diverse dimensions,” JHEP **0008**, 008 (2000) [hep-th/0006062].
 71. E. Witten, “Strong Coupling Expansion Of Calabi-Yau Compactification,” Nucl. Phys. **B471**, 135 (1996) [hep-th/9602070].
 72. J. D. Lykken, “Weak Scale Superstrings,” Phys. Rev. **D54**, 3693 (1996) [hep-th/9603133].
 73. E. Caceres, V. S. Kaplunovsky and I. M. Mandelberg, “Large-volume string compactifications, revisited,” Nucl. Phys. **B493**, 73 (1997) [hep-th/9606036].
 74. N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “The hierarchy problem and new dimensions at a millimeter,” Phys. Lett. **B429**, 263 (1998) [hep-ph/9803315].
 75. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “New dimensions at a millimeter to a Fermi and superstrings at a TeV,” Phys. Lett. **B436**, 257 (1998) [hep-ph/9804398].
 76. L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].
 77. L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. **83**, 4690 (1999) [hep-th/9906064].